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A NEW APPROACH TO CALCULATE TRIGONOMETRIC VALUES

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ABSTRACT

Sinusoidal functions are one of the most basic and essential tools of STEM branch. Every student from his secondary standard and onwards experiences their necessity recurrently. Motive of this article is to enlighten students and researchers about an equation which provides satisfactory cosine values. Method is based upon the linear interpolation for an interval of 5º. Obtained values are highly close to their natural values and hence, are much helpful in rough estimation of scientific and engineering problems in the absence of trigonometric tables and calculator. Purely analytical approach adapted to obtain this equation and have no any derivational background. Maximum deviation of obtained values from their sinusoidal values is 0.0915 only and hence it can be exceedingly helpful if change in 1º hardly matters. Once cosine value get determined, other trigonometric values may also be deduced.

KEYWORDS: Arithmetic Progression (A. P), Greatest Integer Function [X], Linear Interpolation and Reminder Operator (a % b)

INTRODUCTION

Structure of today's trigonometry is based on pillars of work of many mathematicians belonging to different continents, distinct civilizations and for a broad time span. It dates back to the early ages of Egypt and Babylon about 1500 BC. Trigonometry was then forwarded by the Greek astronomer Hipparchus who put a trigonometry table that measured the length of the chord subtending the various angles in a circle of a fixed radius r. His work was taken further by Ptolemy by creating the table of chords with increment of 1° which was known as Menelaus's theorem. Angle measurement in degree was initiated by Babylonians and we are using it till now. Around the same period, Indian mathematicians created the new trigonometry system based on the cosine function instead of the chords. Trigonometry also includes remarkable contribution of Muslim astronomers who compiled both the studies of the Greeks and Indians in the middle age. From13th century and onwards, modern trigonometry was upgraded by Europeans like Isaac Newton, Euler by defining trigonometry functions as ratios rather than lengths of lines. The application of trigonometry came about primarily due to the purposes of astronomy, surveying, navigation, construction and now it has prolonged so far [1]. Siddhantas and Aryabhatia (work of Indian Mathematicians) contain the initial surveying table of cosine values (jyā table) in 3°45' from 0° to 90°, to an accuracy of 4 decimal places [2]. Investigations were made on circle of radius 3438 units. Later on Bhaskara in 7th century provided a formula to obtain cosine values with a relative error less than 1.8% [3] [4].

Purpose

We all are very much familiar about multidisciplinary nature of trigonometry. It plays a vital role in clearing up diverse complications of STEM field. Sometimes, we are known to the perfect procedure to find out solution but problem get unsolved due to lack of trigonometric values. There are several tactics to obtain cosine value of an angle very

accurately with high precision like Tayler series, Bhaskara II formula and many other trigonometric identities. But it becomes inconvenient due their quite long procedure, for example to obtain sum of first 3 or 4 terms of Madhava's cosine series. Mugging up these many formulae may also lead to confusion and tedious to deduce values from them. Equation provided in article, obtained analytically, provides very good approximation. It presents cosine value of angles which are very close to their natural values. Consequently other trigonometric ratios can also be deduces quickly for known cosine values. I hope that it would be fun learning mathematics for students and research scholars.

Method Applied

Purely analytical manner is adapted for the augmentation of formula. On considering the difference between two consecutive cosine values for angles being multiple of 5, a pattern is obtained which has consecutive differences in which cosine values are in pair-wise A.P amazingly! But a small deviation occurs at beginning and at end.

From the $\theta = 20°$ and onwards, a pair wise A.P get formed,

In this pair wise AP 4, 4 is first term, 5, 5 is second term and so on. As 4, 4 is first term occurs for $\theta = 20°$ and $\theta = 25°$.

So, Number of term (n) can be expressed as-

$n = [(\theta\text{-}10)/10]$

Where, [X] is Greatest integer value which is smaller or equal X

Sum of first n terms (S_n) of an AP is given as

$S_n = n/2 *[2a_1+ (n-1)*d]$ (2)

Where, n: number of term, a_1: First term, d: Common difference

Sum of terms which are not in AP is 0.06

So let $a_1 = 0.06$

For the obtained series we have

$a_1= 0.06$ and $d= 0.02$

So, equation (2) reduces to,

$S_n = n/2*[2*0.06+ (n-1)*0.02]$

$= n*[5+n]*10^{-2}$

$= k*10^{-2}$ // Where $k = n*[5+n]$

$Y (\theta) = (100- K) *10^{-2}$

Addition of Correction Terms

The above mentioned equation for S_n could provide very accurate values for angles in between 20° to 75°. So, two correction terms, one of the terms corrects sinusoidal values till 15° and another term corrects values from 80° to 90°, are made added to make it applicable for all angles. For $\theta \leq 15°$, term $[(20\text{-} \theta)/5]*[60/ (40+ \theta)]*10^{-2}$ is added which provides

required corrective values for $\theta \leq 15°$ only and 0 for other angles. Here, term [(20- θ)/5] provides the corrective value while term [45/ (θ+30)] restrict it to work only for $\theta \leq 15°$. Similarly $78° \leq \theta \leq 90°$, term $[2^{(\theta-80)/5}]*10^{-2}$ is added which provides required corrective values for $80° \leq \theta \leq 90°$ and 0 for other angles.

So equation becomes,

$Y(\theta) = \{1 - S_n \pm \text{correction terms}\}$

$Y(\theta) = \{100 - [(\theta-1)/10] * (5 + [(\theta-10)/10]) – (\theta \%2)*(2+ [\theta/10])$

$+ [2^{[(\theta-80)/5]}] – [(20-\theta)/5]* [45/ (\theta+30)]\} *10^{-2}$

Where, % is reminder operator and θ is multiple of 5.

Generalization for All Angles

From our previous exercises, we are familiar with the obtained pattern. Difference in values occurs in pair wise A.P for an interval of 5° angle. As for every interval, difference varies in A.P, so we can conveniently apply linear interpolation to deduce values in between them.

For any value of θ, break it in two components as

$\theta = 5p + q$

Where, p is a positive integer and $0 \leq q \leq 4$

Let, $Y(\theta) = Y(5p) + D(\theta)$

Where, $D(\theta) = q*(Y(5p+5)-Y(5p))/5$

So, $Y(\theta) = (1-q/5)\, Y(5p) + q*(Y(5p+5))/5$

RESULTS

Values obtained by equations are very close to cosine values with high accuracy. Approximate cosine values can be deduced for every angle.

Obtained equation is as,

$$Y(\theta) = (1-q/5)* Y(5p) + q*(Y(5p+5))/5 \quad (1)$$

For any general value of θ

Where $5p \leq \theta < 5p+5$

And,

$$Y(\theta) = \{100 - [(\theta-1)/10] * (5 + [(\theta-10)/10]) – (\theta \%2)*(2+ [\theta /10])$$

$$+ [2^{[(\theta-80)/5]}] – [(20-\theta)/5]* [45/ (\theta+30)]\} *10^{-2} \quad (2)$$

For θ multiple of 5

One example is solved below:

$\theta = 37 = 5*7 + 2$ ($\theta = 5p + q$ form)

So p = 7 and q = 2

As Y (θ) = (1-q/5) Y (5p) + q*(Y (5p+5))/5

On putting values of θ, p and q,

Y (37) = (1-p/5) Y (35) + q*(Y (40))/5

= 0.6*Y (35) + 0.4*(Y (40))

On putting values of Y (35) and Y (40) from above we get,

Y (37) = 0.6*0.81 + 0.4*0.76

= 0.79

Table 1

θ	[(θ -10)/10]	$[2^{[(\theta-80)/5]}]$	[(20-θ)/5] *[45/(θ+30)]	[θ /10]	(θ %2)	Y(θ)	sin θ	Deviation = sin θ - Y(θ)
0	-1	0	4	0	0	1.00	1	0
5	-1	0	3	0	1	0.99	0.996194	0.006194
10	0	0	2	1	0	0.98	0.984807	0.004807
15	0	0	1	1	1	0.96	0.965925	0.005925
20	1	0	0	2	0	0.94	0.939692	-0.00307
25	1	0	0	2	1	0.90	0.906307	0.006307
30	2	0	0	3	0	0.86	0.866025	0.006025
35	2	0	0	3	1	0.81	0.819152	0.009152
40	3	0	0	4	0	0.76	0.766044	0.006044
45	3	0	0	4	1	0.70	0.707106	0.007106
50	4	0	0	5	0	0.64	0.642787	0.002787
55	4	0	0	5	1	0.57	0.573576	0.003576
60	5	0	0	6	0	0.50	0.5	0
65	5	0	0	6	1	0.42	0.422618	0.002618
70	6	0	0	7	0	0.34	0.34202	0.00202
75	6	0	0	7	1	0.25	0.258819	0.008819
80	7	1	0	8	0	0.17	0.173648	0.003648
85	7	2	0	8	1	0.08	0.087155	0.007156
90	8	4	0	19	0	0	0	0

Values of every individual term of expression Y (θ) is mentioned in various columns. Obtained values Y (θ) and cosine values are very close to each other. Y (θ) is obtained by the equation as follows:

Y (θ) = {100 - [(θ-10)/10] * (5 + [(θ-10)/10]) – (θ %2)*(2+ [θ /10])

+ $[2^{[(\theta-80)/5]}]$ – [(20-θ)/5]* [45/ (θ+30)]} *10 (2)

For θ multiple of 5

Accuracy is more than 99% for most of times.

GRAPHS

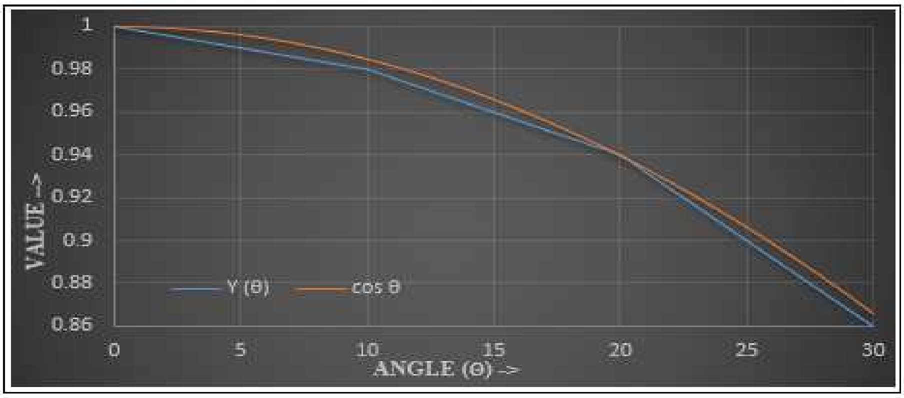

Figure 1

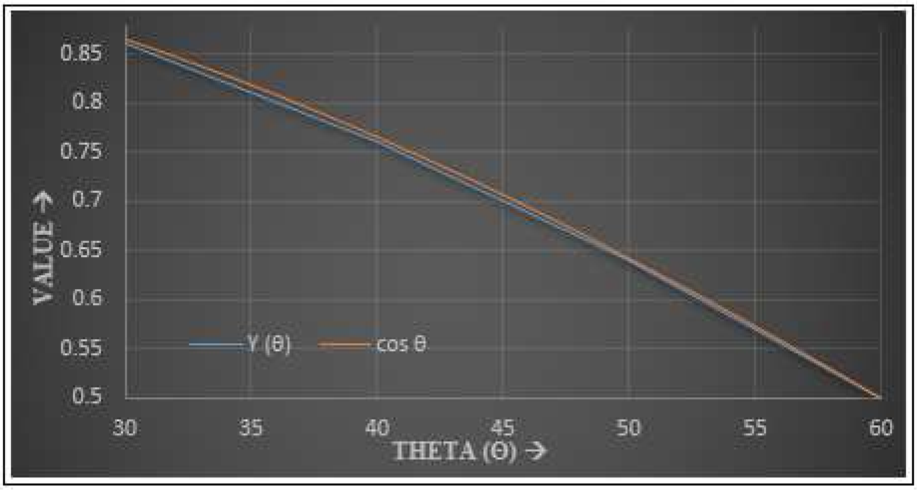

Figure 2

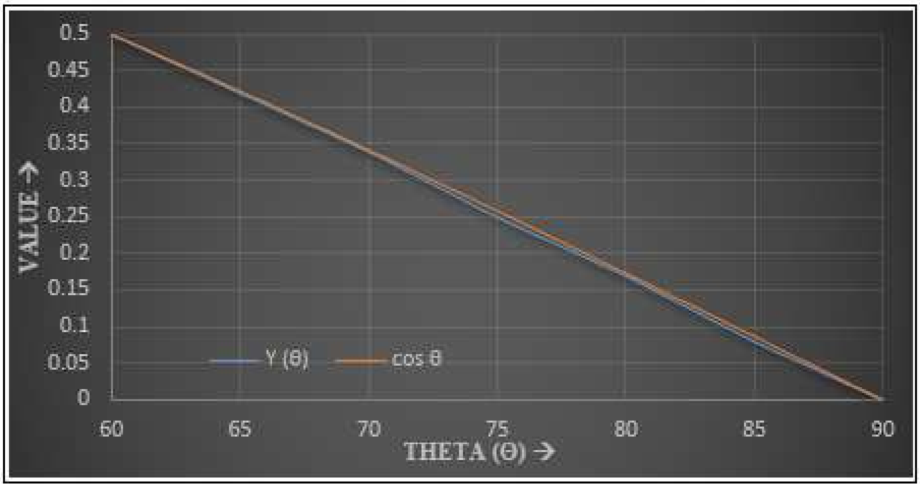

Figure 3

Figure 1, 2, 3

Graphs are potted between angle (on X – axis in degree) and their corresponding 'Natural cosine value' and 'obtained value' (on Y – axis). Natural cosine values are indicated by blue line and obtained values are indicated by red line. In figure 3 both of lines coincides for most of angle. Both lines are very closer to each other in every graph.

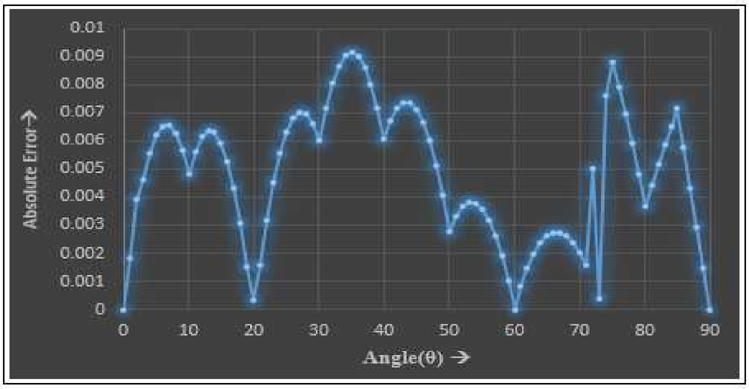

Figure 4

Figure 4

Graph is potted between angle (on X – axis) and corresponding difference between 'natural cosine value' and 'obtained value' (on Y – axis). Deviation of obtained value from standard cosine value is indicated by points. Maximum deviation is nearly 0.0091 only. It is very close to its natural cosine value 0.82903757! It is the maximum deviation found by equation.

CONCLUSIONS

In this paper we deduced an empirical formula which provides approximate cosine values. In the absence of trigonometric table or calculator, this formula is very helpful for a closer and slightly rough estimation of elementary science and engineering problems.

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